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DAM-BREAK FLOWS OVER A BOTTOM DROP

V. V. Ostapenko

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The single-layer shallow-water model is used to study flows generated by dam break over a bed level discontinuity in the form of a drop from which water flows. Emphasis is given to submerged regimes in which downstream wave processes affect the upstream flow. The paper considers solutions in which the total flow energy is conserved on the drop and solutions in which the energy is lost on the drop.

Key words: *shallow water, dam break, bottom drop.*

1. Formulation of the Problem. In the case of a rectangular channel of constant width and varying depth, the single-layer shallow-water differential equations [1–3] ignoring friction are written as

$$h_t + q_x = 0, \qquad q_t + (qv + h^2/2)_x = -hb_x, \tag{1.1}$$

where $h(x,t)$, $q(x,t)$, and $v = q/h$ are the flow depth, rate, and velocity, respectively, and $b(x)$ is the bed level. The acceleration of gravity is $g = 1$. For system (1.1), we consider the following problem of decay of an initial level discontinuity $z = b + h$ in water initially at rest:

$$z(x,0) = \begin{cases} z_0, & x > 0, \\ z_1, & x < 0, \end{cases} \qquad z_1 > z_0, \quad v(x,0) = 0 \tag{1.2}$$

over a sudden change in the bed level

$$b(x) = \begin{cases} 0, & x > 0, \\ \delta, & x < 0, \end{cases} \qquad \delta > 0. \tag{1.3}$$

Because $z_1 > z_0$, it follows that for $t > 0$, the fluid in the vicinity of the discontinuity (1.3) flows in the positive direction of the x axis; therefore, in the nomenclature of [4], the discontinuity (1.3) is a bottom drop from which water flows down. Since the classical problem (1.1), (1.2) of decay of an initial discontinuity of water at rest over a horizontal bottom where $b_x = 0$ is generally called the dam break problem [5], we shall call the problem (1.1)–(1.3) the problem of dam break over a bottom drop. The solution of this problem at $x < 0$ will be called flow on the left of the drop and the solution at $x > 0$, flow on the right of the drop; the exact solution on the discontinuity (1.3) at $x = 0 - 0$ will be called flow over the drop and that at $x = 0 + 0$, flow behind the drop.

The problem (1.1)–(1.3) is a particular case of the general problem of decay of an arbitrary discontinuity over a sudden change in bed level. The latter was studied by Alcrudo and Benkhaldon [6], who gave various examples of solution of this problem assuming that the total flow energy is conserved on the discontinuity (1.3). However, they did not study the uniqueness of these solutions nor determined the regions of existence for them. In the particular case of the problem (1.1)–(1.3) where the flow forming on the bottom drop is critical, there is a nonsubmerged regime of head and tail conjugation. This case was studied by Atavin and Vasil'ev [7], who analyzed hydraulic processes occurring during break of the gate of a multichamber shipping lock. Ostapenko [8, 9] studied the unique solvability of the problem (1.1)–(1.3) at $\delta < 0$ for flow over a bed level discontinuity in the form of a step on which water flows. Unlike in [8], where the total flow energy was assumed to be conserved on the bottom step, in [9] both cases of conservation and loss of the total flow energy on the bottom step were explored.

In the present paper, which is a continuation of [8, 9], the solvability of the generalized decay discontinuity problem (1.1)–(1.3) is examined for both cases of conservation and loss of the total flow energy on the bottom

Lavrent'ev Institute of Hydrodynamics, Siberian Division, Russian Academy of Sciences, Novosibirsk 630090.
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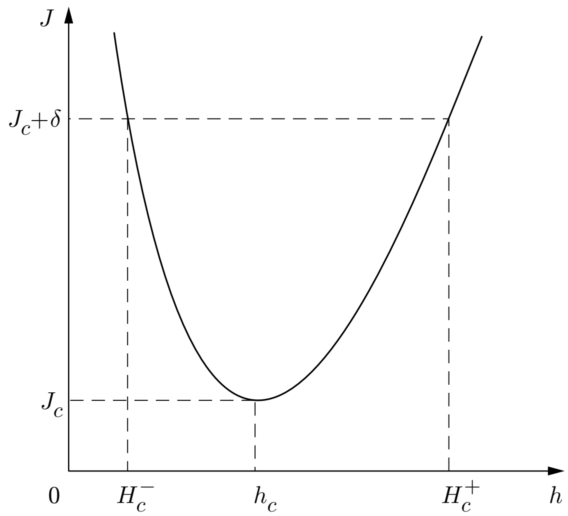

Fig. 1

drop. In contrast to [7], the present study focuses on submerged flow regimes, in which downstream processes affect upstream flow.

2. Admissible Flows over a Bottom Drop with Conservation of the Total Flow Energy. We assume that the total flow energy is conserved on the discontinuity (1.3). Taking into account the continuity of the flow rate, this assumption leads to the following relation on the bottom drop [3, 9]:

$$J(H, q) = J(h, q) + \delta. \tag{2.1}$$

Here the function $J(\xi, q)$ is defined by the formula

$$J(\xi, q) = q^2/(2\xi^2) + \xi, \tag{2.2}$$

$q = hv = HV$ is the flow rate and h and v and H and V are the flow parameters over the drop and behind the drop, respectively. As is shown in [8], condition (2.1) admits two flow configurations on the discontinuity (1.3). For the first configuration,

$$H > h + \delta, \qquad V < v, \qquad V < \sqrt{H}, \qquad v < \sqrt{h} \tag{2.3}$$

and for the second configuration,

$$H < h, \qquad V > v, \qquad V > \sqrt{H}, \qquad v \geqslant \sqrt{h}. \tag{2.4}$$

It follows from Eq. (2.1) that for specified flow rate q and depth H behind the drop, the depth h above the drop is determined as an argument of function (2.2) for which it takes the value $J(H, q) - \delta$. For $q = \text{const}$, function (2.2), whose plot is shown in Fig. 1, reaches a minimum

$$J_c = \min_x J(x, q) = 3q^{2/3}/2$$

for the critical value $h_c = v_c^2 = q^{2/3}$. Therefore, the problem of determination of the depth h has a solution either for $H \geqslant H_c^+(q)$, which corresponds to configuration (2.3), or for $H \leqslant H_c^-(q)$, which corresponds to configuration (2.4), where $H_c^+(q) > H_c^-(q)$ are the arguments of function (2.2), for which it takes the value

$$J_c + \delta = 3q^{2/3}/2 + \delta.$$

For $q > 0$, the critical flow parameters over the drop

$$h_c = q^{2/3}, \qquad v_c = q^{1/3} \tag{2.5}$$

are strictly monotonically increasing functions of q. The following theorem shows that this property holds in shock transitions $h_c \to H_c^{\pm}$ through the discontinuity (1.3).

Theorem 1. *The functions $H_c^{\pm}(q)$ and $V_c^{\pm}(q) = q/H_c^{\pm}(q)$ for $q > 0$ are strictly monotonically increasing functions.*

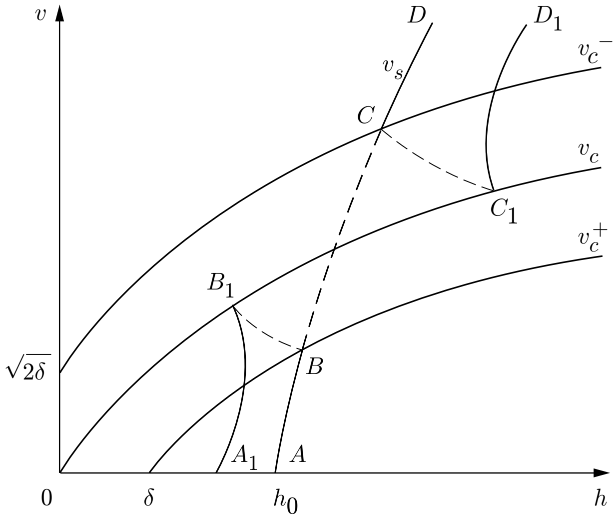

Fig. 2

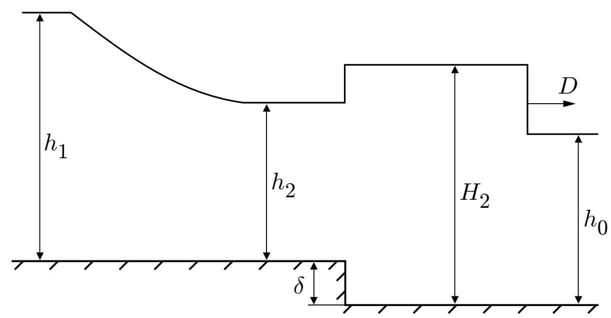

Fig. 3

Proof. The quantities $H_c^{\pm}(q)$ are roots of the equation

$$\frac{q^2}{2H^2} + H = \frac{3}{2}\, q^{2/3} + \delta,$$

whose total differential can be written as

$$\alpha\, dH = \Big(q^{-1/3} - V/H\Big)\, dq, \tag{2.6}$$

where, in view of (2.3) and (2.4),

$$\alpha = 1 - V^2/H = (H - V^2)/H. \tag{2.7}$$

Therefore, using (2.5), from (2.6) we obtain

$$H_q = \frac{1}{\alpha}\Big(\frac{v_c}{h_c} - \frac{V}{H}\Big) = \frac{q(H^2 - h_c^2)}{\alpha H^2 h_c^2}. \tag{2.8}$$

Since $V_q = (q/H)_q = (H - qH_q)/H^2$, taking into account (2.7) and (2.8), we have

$$V_q = \frac{1}{H^2}\Big(H - \frac{q}{\alpha}\Big(\frac{v_c}{h_c} - \frac{V}{H}\Big)\Big) = \frac{\alpha H + V^2 - v_c^2}{\alpha H^2} = \frac{H - h_c}{\alpha H^2}. \tag{2.9}$$

Since $q > 0$, for both admissible configurations (2.3) and (2.4), for the first of which $H > h_c$ and $\alpha > 0$ and for the second, $H < h_c$ and $\alpha < 0$, we obtain $H_q > 0$ and $V_q > 0$ from formulas (2.8) and (2.9). Theorem 1 is proved.

Theorem 1 implies that on the plane of variables (h, v) (Fig. 2), the parameters H and V of the admissible flows behind the drop lie not above the monotonically increasing curve of $v = v_c^+(h) = V_c^+(q_c^+(h))$ or not below the monotonically increasing curve of $v = v_c^-(h) = V_c^-(q_c^-(h))$ [$q_c^{\pm}(H)$ are the inverse functions to $H_c^{\pm}(q)$]; these curves are located in the regions of subcritical and supercritical flows, respectively. As is shown in [8], for $q > 0$, the shock transition $h_c \to H_c^-$ (see Fig. 1) is stable [in Fig. 2, it corresponds to transition from the critical flow line $v = v_c(h) = \sqrt{h}$ to the line $v = v_c^-(h)$] and the shock transition $h_c \to H_c^+$ is unstable [in Fig. 2 corresponds to transition from the line $v = v_c(h)$ to the line $v = v_c^+(h)$].

In the solution of the discontinuity decay problem (1.1)–(1.3), a discontinuous s-wave propagates over the background z_0 to the right of the bottom drop. Behind the front of this wave, the steady flow parameters H_2 and V_2 lie on the shock s-adiabat

$$V = v_s(H, h_0) = \sqrt{(H + h_0)/(2Hh_0)}\,(H - h_0), \qquad h > h_0, \tag{2.10}$$

which is a monotonically increasing function that issues from the point h_0 on the h axis. A centered depression r-wave propagates over the background z_1 to the left of the bottom drop. The steady flow parameters h_2 and v_2 behind this wave lie on the r-wave adiabat

$$v = v_r(h, h_1) = 2(\sqrt{h_1} - \sqrt{h}), \qquad h < h_1, \tag{2.11}$$

which is a monotonically decreasing function that issues from the point h_1 on the h axis.

We assume that the steady flows (H_2, V_2) and (h_2, v_2) are subcritical and, by virtue of this, continue up to the bottom drop (Fig. 3), thus forming a steady discontinuous flow on the drop that satisfies conditions (2.1) and (2.3). In this case, to prove the unique solvability of the problem (1.1)–(1.3) using the generalized adiabatic method [9, 10], it is necessary to examine the conservation of the monotonic properties of the adiabats (2.10) and (2.11) in transition over the bottom drop. In Sec. 3, this question is studied for the shock s-adiabat (2.10).

3. Conservation of the Property of Monotonic Increase of the Function $V(H)$ in Transition over the Bottom Drop. Let us consider a one-parameter family of steady discontinuous flows with depth H and velocity $V(H)$ behind the drop and depth $h(H)$ and velocity $v(H)$ on the drop and establish the conditions under which a monotonic increase in the function $V(H)$ implies unique definiteness and a monotonic increase of the function $v = \tilde{v}(h) = v(H(h))$ [$H(h)$ is the inverse function to $h(H)$].

Theorem 2. *If a positive, strictly monotonically increasing function $V(H)$ with values in the subcritical (supercritical) flow region satisfies the condition $V(H) < v_c^+(H)$ ($V(H) > v_c^-(H)$), its corresponding function $v = \tilde{v}(h)$ with values in the subcritical (supercritical) flow region is uniquely defined and is a strictly monotonically increasing function if and only if the following inequalities are satisfied for the derivative V_H of the function $V(H)$:*

$$\frac{V(H-h)}{H(h-V^2)} < V_H < \frac{h^2 - HV^2}{V(H^2-h^2)}. \tag{3.1}$$

Proof. Since the total differential of Eq. (2.1) can be written as

$$\alpha_0\, dh = \alpha_1\, dH + (V/H - v/h)\, dq, \tag{3.2}$$

where, with allowance for Eqs. (2.2)–(2.4),

$$\alpha_0 = J_h(h,q) = 1 - v^2/h \neq 0, \qquad \alpha_1 = J_H(H,q) = 1 - V^2/H \neq 0, \tag{3.3}$$

then, from (3.2), as in the proof of theorem 1 in [9], we obtain

$$h_H = \frac{1}{\alpha_0}\Big(\alpha_1 + \frac{q(h^2-H^2)}{h^2H^2}\, q_H\Big), \qquad v_H = \frac{(h-H)q_H + \alpha_1 H^2 V_H}{\alpha_0 h^2}. \tag{3.4}$$

Here

$$q_H = (HV)_H = V + HV_H > 0. \tag{3.5}$$

Since $\alpha_1(h - H) < 0$, derivatives (3.4) are not necessarily positive for any positive V_H. Therefore, using formula (3.5), we write them as

$$h_H = (a - bV_H)/(\alpha_0 h^2), \qquad v_H = (cV_H - d)/(\alpha_0 h^2), \tag{3.6}$$

where

$$a = h(h - vV), \qquad b = V(H^2 - h^2), \qquad c = H(h - V^2), \qquad d = V(H - h) \tag{3.7}$$

are positive under conditions (2.3) and negative under conditions (2.4). Assuming that

$$V_H \neq \frac{a}{b} = \frac{h(h-vV)}{V(H^2-h^2)} = \frac{h^2 - HV^2}{V(H^2-h^2)} \quad \Rightarrow \quad h_H \neq 0,$$

from (3.6), we find that

$$v_h = \frac{v_H}{h_H} = \frac{cV_H - d}{a - bV_H} = \alpha\,\frac{V_H - \beta_1}{V_H - \beta_2}. \tag{3.8}$$

Here

$$\alpha = -\frac{c}{b} = \frac{H(V^2-h)}{V(H^2-h^2)} < 0, \quad \beta_1 = \frac{d}{c} = \frac{V(H-h)}{H(h-V^2)} > 0, \quad \beta_2 = \frac{a}{b} = \frac{h^2-HV^2}{V(H^2-h^2)} > 0. \tag{3.9}$$

Let us show that $\beta_2 > \beta_1$, i.e., $ac > bd$. For this, taking into account Eq. (3.7) and the equality $V = hv/H$, we transform the difference $ac - bd$ as follows:

$$\begin{aligned} ac - bd &= H(h - V^2)(h^2 - HV^2) - V^2(H-h)(H^2-h^2) \\ &= (h/H)^2\Big(h(H - v^2)(H^2 - hv^2) - v^2(H-h)(H^2-h^2)\Big). \end{aligned} \tag{3.10}$$

We assume that conditions (2.3) are satisfied. Then, the positiveness of (3.10) follows from the inequalities

$$h > v^2, \qquad H - v^2 > H - h > 0, \qquad H^2 - hv^2 > H^2 - h^2 > 0.$$

If conditions (2.4) are satisfied, then $v^2 > h$ and the positiveness of (3.10) written as

$$ac - bd = (h/H)^2 \Big(h(v^2 - H)(hv^2 - H^2) - v^2(h - H)(h^2 - H^2) \Big)$$

follows from the inequalities

$$h(v^2 - H) > v^2(h - H) > 0, \qquad hv^2 - H^2 > h^2 - H^2 > 0.$$

Since $\beta_2 > \beta_1$, it follows from (3.8) that the derivative v_h is positive only for $\beta_1 < V_H < \beta_2$. If these inequalities are satisfied, derivatives (3.6) are also positive, as follows from

$$h_H = (\beta_2 - V_H)/(\alpha_0 b h^2), \qquad v_H = (V_H - \beta_1)/(\alpha_0 c h^2),$$

where $\alpha_0 b > 0$ and $\alpha_0 c > 0$ for both admissible configurations (2.3) and (2.4). Theorem 2 is proved.

Inequalities (3.1) impose strict restrictions on the value of the derivative of the function $V(H)$ which retains the property of monotonic increase in transition over the drop (1.3) from right to left. Thus, in the subcritical flow region, the boundaries

$$\beta_1 = \frac{V(H - h)}{H(h - V^2)} = \frac{v(H - h)}{H^2 - hv^2}, \qquad \beta_2 = \frac{h^2 - HV^2}{V(H^2 - h^2)} = \frac{h(H - v^2)}{v(H^2 - h^2)} \tag{3.11}$$

of the interval of conservation of monotonicity (β_1, β_2) are, respectively, monotonically increasing and monotonically decreasing functions of V and v and

$$\lim_{V \to 0} \beta_1 = 0, \qquad \lim_{V \to 0} \beta_2 = +\infty, \qquad \lim_{V \to v_c^+ - 0} \beta_i = \lim_{v \to v_c - 0} \beta_i = f(H_c^+, V_c^+).$$

Here with allowance for (2.5), $f(H, V) = q^{1/3}/(H + q^{2/3})$; $q = HV$. Conversely, in the supercritical flow region, the boundaries β_1 and β_2 are, increasing, monotonically decreasing and monotonically functions of V and v, respectively, and

$$\lim_{V \to +\infty} \beta_1 = 0, \qquad \lim_{V \to +\infty} \beta_2 = +\infty, \qquad \lim_{V \to v_c^- + 0} \beta_i = \lim_{v \to v_c + 0} \beta_i = f(H_c^-, V_c^-).$$

From this it follows that in the subcritical flow region, the length of the interval (β_1, β_2), being infinitely great as $V \to 0$, decreases monotonically with increase in V and becomes infinitely small as $V \to v_c^+ - 0$; in the supercritical flow region, the length of this interval, being infinitely great as $V \to +\infty$, decreases monotonically with decrease in V and becomes infinitely small as $V \to v_c^- + 0$. Therefore, in order that a function $V(H)$ with values in the subcritical flow region remain a monotonically increasing function in transition over the drop as $V \to v_c^+ - 0$, its derivative V_H must satisfy the condition

$$\lim_{V \to v_c^+ - 0} V_H = f(H_c^+, V_c^+). \tag{3.12}$$

Similarly, in order that a function $V(H)$ with values in the supercritical flow region remain a monotonically increasing function in transition over the drop as $V \to v_c^- + 0$, its derivative V_H must satisfy the condition

$$\lim_{V \to v_c^- + 0} V_H = f(H_c^-, V_c^-). \tag{3.13}$$

If a monotonically increasing function $V(H)$ is the shock s-adiabat (2.10), the depths $H_c^\pm$ included in conditions (3.12) and (3.13) should be roots of the equation

$$(H + q^{2/3})(v_s)_H = q^{1/3}, \tag{3.14}$$

where

$$q = Hv_s = (H - h_0)\sqrt{\frac{H(H + h_0)}{2h_0}}, \qquad (v_s)_H = \frac{2H^2 + Hh_0 + h_0^2}{2H\sqrt{2Hh_0(H + h_0)}}. \tag{3.15}$$

Since Eq. (3.14), in view of (3.15), is uniform with respect to H and h_0, it can be written as

$$(\xi + \eta^2)(2\xi^2 + \xi + 1) = 2\xi\eta\sqrt{2\xi(\xi + 1)}, \tag{3.16}$$

where $\eta = \sqrt[6]{\xi(\xi + 1)(\xi - 1)^2/2}$ and $\xi = H/h_0$.

We introduce the notation

$$a = 2\xi^2 + \xi + 1, \qquad b = \xi\sqrt{2\xi(\xi+1)}, \qquad c = \xi a = \xi(2\xi^2 + \xi + 1)$$

and view Eq. (3.16) as being quadratic for η:

$$a\eta^2 - 2b\eta + c = 0. \tag{3.17}$$

Equation (3.17) has real roots if its discriminant

$$D = b^2 - ac = 2\xi^3(\xi+1) - \xi(2\xi^2 + \xi + 1)^2 \geqslant 0.$$

However, for $\xi > 1$m, the discriminant

$$D < 2\xi^3(\xi+1) - \xi(4\xi^4 + 4\xi^3) = 2\xi^3(\xi+1)(1-2\xi) < 0.$$

Hence, Eqs. (3.16) and (3.17) do not have real solutions for $\xi > 1$ and Eq. (3.14) does not have real solutions for $H > h_0$. Thus, we proved that the shock s-adiabat (2.10) does not satisfy conditions (3.12) and (3.13) for all $h_0 > 0$ and $\delta > 0$. Hence, for the shock s-adiabats, the property of monotonic increase is lost in transition over the bottom drop. This property is violated when the s-adiabat approaches the critical line $v_c^+(h)$ from below and the critical line $v_c^-(h)$ from above. This is illustrated in Fig. 2, where the monotonic curve $ABCD$ is a plot of the adiabat (2.10) and the nonmonotonic curves A_1B_1 and C_1D_1 are images of the segments AB and CD of this adiabat for transition over the drop.

It follows from Fig. 2 that the monotonicity is violated because when approaching the of critical flow line $v = \sqrt{h}$, the function $v = \tilde{v}(h)$, plotted as the curves A_1B_1 and C_1D_1, becomes nonunique; therefore, the function $v = \tilde{v}(h)$ cannot be used to prove the unique solvability of the discontinuity decay problem (1.1)–(1.3) using the generalized adiabatic method. In this connection, we study the problem of conservation of monotonic decrease for the wave r-adiabat (2.11) in transition over the drop (1.3) from right to left.

4. Conservation of Monotonic Decrease of the Function $v(h)$ in Transition over the Bottom Drop. Let us consider a one-parameter family of steady discontinuous flows with depth h and velocity $v(H)$ on the drop and depth $H(h)$ and velocity $V(h)$ behind the drop and establish the conditions under which a monotonic decrease in the function $v(h)$ implies unique definiteness and a monotonic increase of the function $V = \tilde{V}(H) = V(h(H))$ [$h(H)$ is a the inverse function to $H(h)$].

Theorem 3. *If the values of a positive, strictly monotonically decreasing function $v(h)$ belong to the subcritical (supercritical) flow regions, its corresponding function $V = \tilde{V}(H)$ with values in the subcritical (supercritical) flow region is uniquely defined and is a strictly monotonically increasing function if and only if the following inequalities are satisfied for the derivative v_h of the function $v(h)$:*

$$\frac{hv^2 - H^2}{v(H^2 - h^2)} < v_h < \frac{v(H-h)}{h(v^2 - H)}. \tag{4.1}$$

Proof. Using (3.3), from Eq. (3.2) we obtain the following formulas [similar to (3.4)] for the derivatives H_h and V_h:

$$H_h = \frac{1}{\alpha_1}\Big(\alpha_0 + \frac{q(H^2 - h^2)}{H^2h^2}\, q_h\Big), \qquad V_h = \frac{(H-h)q_h + \alpha_0 h^2 v_h}{\alpha_1 H^2}. \tag{4.2}$$

Here

$$q_h = (hv)_h = v + hv_h. \tag{4.3}$$

Substituting (4.3) into (4.2), after transformation, we obtain

$$H_h = (b_1 v_h + a_1)/(\alpha_1 H^2), \qquad V_h = (c_1 v_h + d_1)/(\alpha_1 H^2), \tag{4.4}$$

where $a_1 = H(H - Vv)$, $b_1 = v(H^2 - h^2)$, $c_1 = h(H - v^2)$, and $d_1 = v(H - h)$ are positive under conditions (2.3) and negative under conditions (2.4). Assuming that

$$v_h \neq -\frac{a_1}{b_1} = \frac{H(H - Vv)}{v(H^2 - h^2)} = \frac{H^2 - hv^2}{v(H^2 - h^2)} \quad \Rightarrow \quad H_h \neq 0,$$

from (4.4), using the notation (3.9), we have

$$V_H = \frac{V_h}{H_h} = \frac{c_1 v_h + d_1}{b_1 v_h + a_1} = \beta_2 \frac{v_h - \gamma_2}{v_h - \gamma_1}, \tag{4.5}$$

where

$$\gamma_1 = \alpha = \frac{H(V^2 - h)}{V(H^2 - h^2)} < 0, \qquad \gamma_2 = \frac{\alpha\beta_1}{\beta_2} = \frac{V(H-h)}{HV^2 - h^2} < 0.$$

Since $\beta_2 > \beta_1$, it follows that $\gamma_2 > \gamma_1$; therefore, formula (4.5) implies that the derivative V_H is negative only for $\gamma_1 < v_h < \gamma_2$. If these inequalities are satisfied, the derivative H_h is positive and the derivative V_h are negative, as follows from

$$H_h = (v_h - \gamma_1)/(\alpha_1 b_1 H^2), \qquad V_h = (v_h - \gamma_2)/(\alpha_1 c_1 H^2),$$

where $\alpha_1 b_1 > 0$ and $\alpha_1 c_1 > 0$ for both admissible configurations (2.3) and (2.4). Since $V = hv/H$, the expressions for the boundaries of the interval of conservation of monotonicity (γ_1, γ_2) can be written as

$$\gamma_1 = \frac{hv^2 - H^2}{v(H^2 - h^2)}, \qquad \gamma_2 = \frac{v(H-h)}{h(v^2 - H)}. \tag{4.6}$$

Theorem 3 is proved.

Inequalities (4.1) impose substantial restrictions on the value of the derivative of the function $v(h)$ which retains the property of monotonic decrease in transition over the drop (1.3) from left to right. Similarly to boundaries (3.11), boundaries (4.6) in the subcritical flow region are monotonically increasing and monotonically decreasing functions of v, respectively, such that

$$\lim_{v\to 0} \gamma_1 = -\infty, \qquad \lim_{v\to 0} \gamma_2 = 0, \qquad \lim_{v\to v_c - 0} \gamma_i = -\frac{1}{\sqrt{h}}.$$

In the supercritical flow region, they are monotonically decreasing and monotonically increasing functions of v, respectively, such that

$$\lim_{v\to +\infty} \gamma_1 = -\infty, \qquad \lim_{v\to +\infty} \gamma_2 = 0, \qquad \lim_{v\to v_c + 0} \gamma_i = -\frac{1}{\sqrt{h}}.$$

From this it follows that in the subcritical flow region, the length of the interval (γ_1, γ_2), being infinitely great as $v \to 0$, decreases monotonically with increase in v and becomes infinitely small as $v \to v_c - 0$; in the supercritical flow region, the length of this interval, being infinitely large as $v \to +\infty$, decreases monotonically with decrease in v and becomes infinitely small as $v \to v_c + 0$. In spite of this, for all $h_1 > 0$, the derivative $(v_r)_h = -1/\sqrt{h}$ of the wave r-adiabat (2.11) satisfies inequalities (4.1) both in the subcritical flow region ($v_r < \sqrt{h}$) and in the supercritical flow region ($v_r > \sqrt{h}$), i.e.,

$$\frac{H^2 - hv_r^2}{v_r(H^2 - h^2)} > \frac{1}{\sqrt{h}} > \frac{v_r(H-h)}{h(H - v_r^2)} \tag{4.7}$$

for all positive $h \neq 4h_1/9$.

To prove inequalities (4.7), we multiply each of them by the velocity $v_r > 0$. As a result, we obtain

$$\frac{H^2 - h^2 f_r^2}{H^2 - h^2} > f_r > \frac{(H-h)f_r^2}{H - hf_r} \tag{4.8}$$

($f_r = v_r/\sqrt{h}$ is the Froude number). Direct check shows that inequalities (4.8) are true both under conditions (2.3), when $f_r < 1$, and under conditions (2.4), when $f_r > 1$.

Thus, in contrast to the shock s-adiabat (2.10), the wave r-adiabat (2.11) remains monotonic in transition over the bottom drop for all $h_1 > 0$ and $\delta > 0$. Therefore, in contrast to the problem of dam break over a step [9], it is the image of the wave r-adiabat that should be used in the generalized adiabatic method to prove the unique solvability of the discontinuity decay problem (1.1)–(1.3) for the case of submerged flow over a drop.

5. Self-Similar Solutions for Submerged Flow over the Drop. As is shown in Sec. 4, the portion of the wave r-adiabat (2.11) lying in the subcritical flow region for $h > 4h_1/9$ (curve h_1P_2 in Fig. 4) is converted by means of the energy relation (2.1) to a monotonically decreasing function $V = \tilde{V}_r(H)$, which also belongs to the subcritical flow region (curve z_1P_4 in Fig. 4). Since relation (2.1) is written as the equation

$$H^3 - (v^2/2 + h + \delta)H^2 + q^2/2 = 0,$$

which is cubic for H, the function $V = \tilde{V}_r(H)$ can be defined in parametric form

$$H = H_r(h) = F(h, v_r(h, h_1)), \quad V = V_r(h) = hv_r(h, h_1)/H_r(h), \quad h \in (4h_1/9, h_1), \tag{5.1}$$

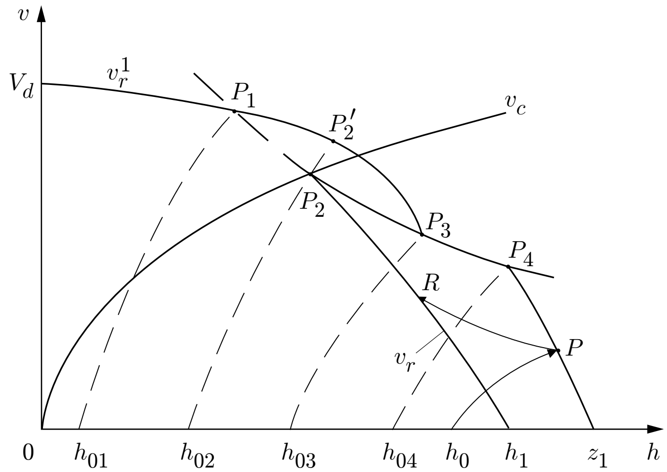

Fig. 4

where $v_r(h, h_1)$ is the wave r-adiabat (2.11); the function $F(h, v)$ is defined by Cardano's formula

$$F(h,v) = a\Big(2\cos\Big(\frac{1}{3}\arccos\Big(1 - \frac{h^2v^2}{4a^3}\Big)\Big) + 1\Big), \qquad a = \frac{v^2 + 2(h+\delta)}{6}.$$

From a monotonic increase in the shock s-adiabat (2.10) and a monotonic decrease in the function $V = \tilde{V}_r(H)$, which is the image of the wave r-adiabat (2.11) in transition over the drop, it follows that the discontinuity decay problem (1.1)–(1.3) is uniquely solvable for

$$h_0 \in (h_{04}, z_1), \qquad z_1 = h_1 + \delta. \tag{5.2}$$

Here h_{04} is the point on the h axis for which the shock s-adiabat $v_s(h, h_{04})$ intersects the point P_4 in Fig. 4.

The value of h_{04} is calculated as follows. First, the formulas

$$h_c = 4h_1/9, \qquad v_c = \sqrt{h_c} = 2\sqrt{h_1}/3 \tag{5.3}$$

are used to determine the coordinates of the point P_2 at which the wave r-adiabat (2.11) intersects the critical flow line $v = \sqrt{h}$. Next, from (5.1) and (5.3) we derive the formulas

$$H_c^+ = H_r(h_c), \qquad V_c^+ = V_r(h_c) = h_c^{3/2}/H_r(h_c),$$

and use them to find the coordinates of the point P_4 at which line (5.1) intersects the hyperbola

$$v = q_c/h, \qquad q_c = h_c v_c = 8\sqrt{h_1^3}/27, \tag{5.4}$$

issuing from the point P_2. After this, the value of h_{04} is determined from the equation

$$v_s(H_c^+, h_{04}) = V_c^+, \tag{5.5}$$

where $v_s(h, h_0)$ is a function of the shock s-adiabat (2.10). Equation (5.5) can be written as

$$\xi^3 - \xi^2 - (2(V_c^+)^2/H_c^+ + 1)\xi + 1 = 0,$$

which is a cubic equation for $\xi = h_{04}/H_c^+$. Solution of this equation yields $h_{04} = G(H_c^+, V_c^+)$, where the function $G(h, v)$ is calculated from Cardano's formula

$$G(h,v) = \frac{h}{3}\Big(2p\cos\Big(\frac{1}{3}\Big(2\pi - \arccos\frac{9f^2 - 8}{p^3}\Big)\Big) + 1\Big) \tag{5.6}$$

($p = \sqrt{6f^2 + 4}$ and $f = v/\sqrt{h}$).

In the exact solution obtained under condition (5.2) there are two regions of constant subcritical flows (see Fig. 3): 1) the region (h_2, v_2) between the depression r-wave and the drop; 2) the region (H_2, V_2) between the drop and the discontinuous s-wave. The constant-flow parameters H_2 and V_2 are the coordinates of the point P at which the shock s-adiabat (2.10) intersects the plot of function (5.1) (curve z_1P_4 in Fig. 4), and the constant-flow parameters h_2 and v_2 are the coordinates of the point R at which the wave r adiabat (2.11) intersects the hyperbola

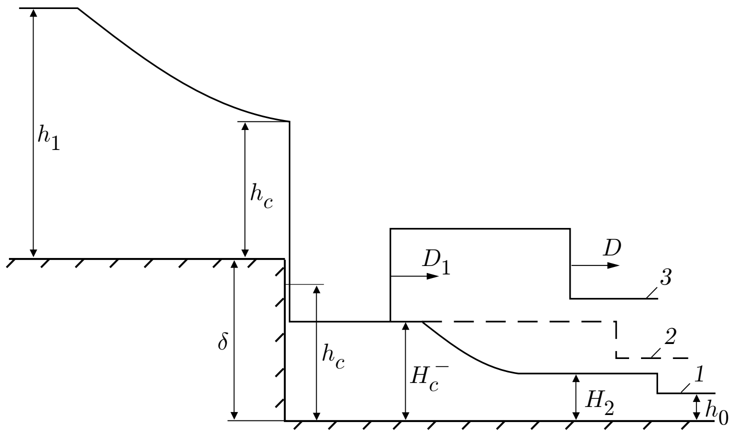

Fig. 5

$v = q/h$, $q = H_2V_2$ issuing from the point P. The depths h_2 and H_2 and the velocities v_2 and V_2 of these constant flows satisfy the system of equations

$$v_2 = v_r(h_2, h_1), \quad V_2 = v_s(H_2, h_0), \quad V_2^2/2 + H_2 = v_2^2/2 + h_2 + \delta, \quad H_2V_2 = h_2v_2,$$

whose solution can be obtained as follows. First, from the equation $V_r(h_2) = v_s(H_r(h_2), h_0)$, where $H_r(h)$ and $V_r(h)$ are functions (5.1), we find the depth h_2 on the left of the drop. Next, the flow velocity v_2 on the left of the drop is determined from the formula $v_2 = v_r(h_2, h_1) = 2(\sqrt{h_1} - \sqrt{h_2})$ and the flow parameters H_2 and V_2 on the right from the drop are calculated from the formulas $H_2 = H_r(h_2)$ and $V_2 = V_r(h_2)$. After that, using the formula

$$D = \sqrt{H_2(H_2 + h_0)/(2h_0)}, \tag{5.7}$$

we find the speed of propagation of the discontinuous s-wave. Formula (5.7) follows from the Hugoniot conditions

$$D[h] = [q], \qquad D[q] = [qv + h^2/2] \tag{5.8}$$

obtained from the laws of conservation of mass and total momentum (1.1) over a horizontal bottom when $b_x = 0$.

As follows from Fig. 4, with decrease in the initial depth h_0 from z_1 to h_{04}, the depth H_2 behind the drop decreases from z_1 to H_c^+ and the velocity behind the drop (V_2) increases from 0 to V_c^+; at the same time, the depth h_2 on the drop decreases from h_1 to h_c, and the velocity v_2 on the drop increases from 0 to v_c. Thus, the solutions constructed are flows with a submerged bottom drop since the flow parameters H_2 and V_2 behind the drop affect the flow parameters h_2 and v_2 on the drop; in this case, the flow level always increases when the fluid flows down the drop (see Fig. 3). Thus, in the case of the submerged flow regime that occurs under condition (5.2), the discontinuity decay problem (1.1)–(1.3) is uniquely solvable under the assumption (2.1) of conservation of the total flow energy on the bottom drop.

6. Self-Similar Solutions for Nonsubmerged Flow over the Drop. We assume that the initial depth h_0 to the right of the drop satisfies the inequality $h_0 < h_{04}$. By virtue of this, the right boundary of the depression r-wave propagating over the background z_1 is located directly on the drop (Fig. 5), thus generating critical flow (h_c, v_c) over the drop, whose parameters are calculated from formulas (5.3). In this case, two characteristics of system (1.1) arrive at the drop (1.3) from the left (at $x \leqslant 0 - 0$), producing a nonsubmerged flow regime in which the flow parameters behind the drop have no effect on the flow over the drop.

As is shown in [8], under the assumption (2.1) of conservation of the total flow energy in transition over the drop, the solutions with the critical flow over the drop are stable only under conditions (2.4), where the flow behind the drop is supercritical. The parameters H_c^- and V_c^- of this supercritical flow do not depend on the wave processes on the right of the drop at $x > 0$ and are completely determined by the critical flow (h_c, v_c) over the drop. In view of (2.1) and (5.3), the values of H_c^- and V_c^- satisfy the energy relation

$$(V_c^-)^2/2 + H_c^- = 3h_c/2 + \delta,$$

which, with allowance for

$$H_c^- V_c^- = H_c^+ V_c^+ = h_c v_c = q_c, \tag{6.1}$$

is conveniently written as $(V_c^-)^3 - (3h_c + 2\delta)V_c^- + 2q_c = 0$, which is cubic for V_c^-. Solving this equation, we obtain $V_c^- = \Psi(h_c, q_c)$ and $H_c^- = q_c/V_c^-$, where the function $\Psi(h,q)$ is calculated from Cardano's formula

$$\Psi(h,q) = 2r\cos\Bigl(\frac{1}{3}\arccos\Bigl(-\frac{q}{r^3}\Bigr)\Bigr), \qquad r = \sqrt{h + \frac{2}{3}\delta}.$$

In Fig. 4, the flow (H_c^-, V_c^-) corresponds to the point P_1 on the part of the hyperbola (5.4) shown by a dashed line that issues from the point P_2 into the region of supercritical flows.

Construction of a solution of the problem on the right of the drop at $x > 0$ reduces solving the classical problem of decay of a discontinuity above a horizontal bottom [11] for system (1.1) with the following initial data:

$$h(x,0) = \begin{cases} h_0, & x > 0, \\ H_c^-, & x \leqslant 0, \end{cases} \qquad v(x,0) = \begin{cases} 0, & x > 0, \\ V_c^-, & x \leqslant 0. \end{cases} \tag{6.2}$$

To solve problem (6.2), it is necessary to find the point intersection of the monotonically increasing shock s-adiabat (2.10) issuing from the point h_0 on the h axis and the monotonically decreasing r-adiabat

$$v = v_r^1(h, H_c^-, V_c^-) = V_c^- + \alpha(h, H_c^-), \qquad \alpha(h,H) = \begin{cases} -v_s(h,H), & h \geqslant H, \\ v_r(h,H), & h \leqslant H, \end{cases} \tag{6.3}$$

issuing from the point $P_1 = (H_c^-, V_c^-)$.

We first assume that $h_0 \in (0, h_{01})$ [$h_{01} = G(H_c^-, V_c^-)$ is the point on the h axis for which the shock s-adiabat $v_s(h, h_{01})$ passes through the point P_1]. Then, the s-adiabat (2.10) intersects the r-adiabat (6.3) in its wave part (on the left of the point P_1 in Fig. 4), thus forming flow on the right of the drop with a depression r-wave and a discontinuous s-wave (line 1 in Fig. 5). The depth H_2 of the constant flow (H_2, V_2) between these waves is determined from the equation $v_s(H_2, h_0) - v_r(H_2, H_c^-) = V_c^-$, after which the velocity V_2 of this flow is calculated from the formula

$$V_2 = v_s(H_2, h_0), \tag{6.4}$$

and the speed of propagation D of the discontinuous s-wave is found from formula (5.7). If $h_0 = 0$, the discontinuous s-wave degenerates and the right boundary of the decreasing r-wave located on the right of the drop moves over the dry channel with the velocity $V_d = V_c^- + 2\sqrt{H_c^-}$. If $h_0 = h_{01}$, the depression r-wave degenerates on the right of the drop and the constant supercritical flow (H_c^-, V_c^-) reaches the front of the discontinuous s-wave (dashed line 2 in Fig. 5), i.e., in this case, $H_2 = H_c^-$ and $V_2 = V_c^-$.

We now assume that $h_0 \in (h_{01}, h_{05})$ [$h_{05} = G(H_c^-, -V_c^-)$ is the coordinate of the point at which the r-adiabat (6.3) intersects the h axis). Then, the s-adiabat (2.10) intersects the r-adiabat (6.3) in its shock part (on the right of the point P_1 in Fig. 4), thus forming flow with two discontinuous waves. The depth H_2 of the constant flow (H_2, V_2) between these waves is determined from the equation $v_s(H_2, h_0) + v_s(H_2, H_c^-) = V_c^-$. After that, the velocity V_2 of this flow and the speed D of propagation of the discontinuous s-wave are calculated from formulas (6.4) and (5.7), and the speed D_1 of propagation of the discontinuous r-wave is calculated from the formula

$$D_1 = V_c^- - \sqrt{H_2(H_2 + H_c^-)/(2H_c^-)} \tag{6.5}$$

derived from the Hugoniot conditions (5.8).

Within the framework of the general problem (1.1)–(1.3), the solution with two discontinuous waves on the right of the drop (line 3 in Fig. 5) is meaningful only if the speed of the discontinuous r-wave $D_1 > 0$, which is equivalent to the condition $H_2 < H_2^*$, where

$$H_2^* = H_c^-(\sqrt{1 + 8(V_c^-)^2/H_c^-} - 1)/2$$

is a root of equation (6.5) for $D_1 = 0$. At the depth $H_2 = H_2^*$, the discontinuous r-wave merges with the discontinuity on the drop, thus forming a uniform standing jump, behind which the velocity $V_2^* = q_c/H_2^*$. This fact and Eq. (6.1) imply that the point $P_3 = (H_2^*, V_2^*)$ is the point of intersection of the shock part of the r-adiabat (6.3) (curve $P_1P_2'P_3$ in Fig. 4) with the part of the hyperbola (5.4) located in the subcritical flow region (line P_2P_4 in Fig. 4). In this case, the point $P_3 = (H_2^*, V_2^*)$ lies on the hyperbola (5.4) between the points $P_2 = (h_c, v_c)$ and $P_4 = (H_c^+, V_c^+)$ since on the hydraulic jump, the total flow energy is lost, whereas in the shock transition $h_c \to H_c^+$ (transition from the point P_2 to the point P_4 in Fig. 4), it is conserved.

Thus, the solution with two discontinuous waves on the right of the drop exists only under the condition $h_0 \in (h_{01}, h_{03})$, in which $h_{03} \in (h_{02}, h_{04})$ [$h_{02} = G(h_c, v_c)$ and $h_{03} = G(H_2^*, V_2^*)$ are points on the h axis that are

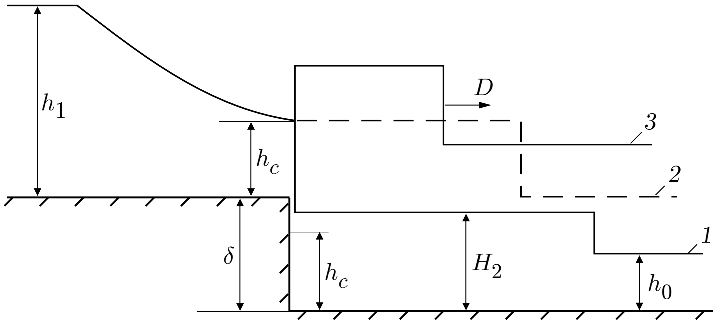

Fig. 6

the start points for the shock s-adiabats (2.10) passing through the points P_2 and P_3 in Fig. 4, respectively]. From this it follows that nonsubmerged flow regimes in which the total flow energy is conserved in transition over the drop can exist only at $h_0 < h_{03} < h_{04}$. This implies that at the initial downstream depths

$$h_0 \in (h_{03}, h_{04}), \tag{6.6}$$

the discontinuity decay problem (1.1)–(1.3) is unsolvable under the assumption (2.1) of conservation of the total flow energy in transition over the bottom drop. This is a fundamental feature that distinguishes this problem from the problem of discontinuity decay (1.2) over a bottom step [for $\delta < 0$ in formula (1.3)], which, as is shown in [9], is always uniquely solvable under condition of conservation of the total flow energy in transition over a step.

7. Energetically Stable Solutions with Three Characteristics that Arrive at the Bottom Drop. Since, under condition (6.6), the discontinuity decay problem (1.1)–(1.3) is unsolvable using the energy relation (2.1), we consider its solutions in which the total flow energy is lost in transition over the bottom drop. As is shown in [9], there are two classes of such solutions: solutions in which two characteristics of system (1.1) arrive at the discontinuity (1.3) and solutions in which three characteristics of this system arrive at the discontinuity (1.3). To close the shallow-water model for solutions of the first class, one needs to modify Eq. (2.1) by introducing an heuristic parameter that defines the part of the total flow energy that is lost in transition over the drop. For solutions of the second class, it suffices to require continuity of the flow rate $[q] = 0$ to close the conditions on the discontinuity (1.3). In this case, the part of the total flow energy that is lost during transition over the drop is determined uniquely within shallow-water theory without introducing any heuristic parameters. In this paper, we consider solutions of only the second class.

In order that three characteristics arrive at the discontinuity (1.3) in the solution of the problem (1.1)–(1.3), the right boundary of the depression r-wave propagating over the background z_1 must be located on the drop to form critical flow (h_c, v_c) on it and the flow behind the drop (H_2, V_2) must be subcritical or supercritical, i.e., $V_2 \leqslant \sqrt{H_2}$, where $H_2 \geqslant h_c$ (Fig. 6). Such discontinuous solutions over the drop are in a sense similar to discontinuous waves over an even bottom, whose characteristic stability requires three characteristics to arrive at the wave front and only one of them to leave the front [11]. The presence of the third arriving characteristic for discontinuous waves allows one to uniquely determine the speed of their propagation and, for the case of immovable discontinuities over the drop, the energy losses on the drop.

As is known [11], the nongrowth of the total energy on discontinuous solutions of the shallow-water equation (1.1) can be used as an alternative criterion of their stability, which is an analogue of the entropy criterion of stability for shock waves in gas dynamics [12]. This criterion was extended to the general theory of hyperbolic systems [13]. However, in contrast to discontinuous waves, for which the characteristic and energy (entropy) criteria of their stability are equivalent [11, 14], in the case of discontinuous solutions of the second class considered in the present paper, the energy stability criterion significantly limits the set of these solutions. Indeed, if the critical flow (h_c, v_c) has formed on the drop, the subcritical flow parameters behind the drop H_2, V_2 $(H_2V_2 = h_cv_c = q_c)$ can correspond to any point of the part of the hyperbola (5.4) located in the subcritical flow region. However, to ensure nongrowth in the total flow energy in transition over the drop, the point (H_2, V_2) must lie between the lines v_c and v_c^+ in Fig. 2, i.e., between the points P_2 and P_4 of the hyperbola (5.4) in Fig. 4. This follows from the energy criterion of stability that is a consequence of the law of conservation of the total energy (see [8, 11])

$$(qv + h^2/2)_t + (q(v^2/2 + h))_x = -qb_x.$$

At an immovable discontinuity over the drop, this criterion has the form

$$q_c[v^2/2 + z] \leqslant 0. \tag{7.1}$$

Since $q_c > 0$, from (7.1), we have

$$J(H_2, q_c) \leqslant J(h_c, q_c) + \delta = J_c + \delta, \tag{7.2}$$

where the function $J(\xi, q)$ is defined by formula (2.2). From the plot of this function (see Fig. 1), one can see that for the subcritical flow (H_2, V_2), condition (7.2) is equivalent to the inequalities $h_c \leqslant H_2 \leqslant H_c^+$ and $V_c^+ \leqslant V_2 \leqslant v_c$, which imply that the point with the coordinates H_2 and V_2 lies on the hyperbola (5.4), between the points $P_2 = (h_c, v_c)$ and $P_4 = (H_c^+, V_c^+)$. Thus, energetically stable solutions of the second class exists at the initial downstream depths

$$h_0 \in [h_{02}, h_{04}], \tag{7.3}$$

when shock s-adiabat (2.10) intersects the hyperbola (5.4) between the points P_2 and P_4 in Fig. 4. In these solutions (see Fig. 6), the depth of the constant flow (H_2, V_2) between the bottom drop and the discontinuous s-wave is determined from the equation $H_2 v_s(H_2, h_0) = q_c$. After that, the flow velocity is calculated from the formula $V_2 = q_c/H_2$ and the velocity of the discontinuous s-wave, from formula (5.7).

It follows from Fig. 4 that with increase in the initial depth h_0 from h_{02} to h_{04}, the depth H_2 behind the drop increases monotonically and continuously from h_c to H_c^+, whereas the velocity V_2 behind the drop decreases monotonically and continuously from v_c to V_c^+. This fact and the inequality $H_c^+ > h_c + \delta$ imply that there exists an initial depth $h_0' \in (h_{02}, h_{04})$ such that the flow levels on the drop and behind the drop are identical, i.e., $h_c + \delta = H_2$ (dashed line 2 in Fig. 6). The value of h_0' is found from the equation $v_s(H_c, h_0') = V_c$, where $H_c = h_c + \delta$ and $V_c = q_c/H_c$. Solution of this equation using formula (5.6) yields $h_0' = G(H_c, V_c)$. For $h_0 \in (h_{02}, h_0')$, the flow level (H_2, V_2) behind the drop is lower than the critical flow level (h_c, v_c) on the drop, i.e., $H_2 < h_c + \delta$ (line 1 in Fig. 6), whereas for $h_0 \in (h_0', h_{04})$, conversely, $H_2 > h_c + \delta$ (line 3 in Fig. 6).

From the adiabatic diagram shown in Fig. 4 it follows that the discontinuity decay problem (1.1)–(1.3) is solvable at all $h_0 \in [0, z_1)$ either for flows for which the total flow energy is conserved on the drop or for flows of the second class in which the total flow energy is lost on the drop. However, this problem is solved nonuniquely because for each initial depth $h_0 \in [h_{02}, h_{03})$, there exist two solutions: a second-class solution with one discontinuous wave and subcritical flow behind the drop (see Fig. 6), which corresponds to the intersection of the s-adiabat (2.10) and the hyperbola (5.4) on the section P_2P_3 (in this solution, the energy is lost on the drop), and a solution with two discontinuous waves and supercritical flow behind the drop (line 3 in Fig. 6), which corresponds to the intersection of the s-adiabat (2.10) and the shock part of the r-adiabat (6.3) on the section $P_2'P_3$ (in this solution, the total energy is conserved on the drop). Which of these flow regimes occurs in practice depends on the particular conditions of laboratory experiments.

Let us consider in more detail the boundaries of the segment (7.3), on which second-class solutions are energetically stable. The continuously increasing initial depth h_0 passes through the right boundary h_{04} of the segment (7.3) (in Fig. 4, this corresponds to the continuous passage of the point P from the hyperbola P_2P_4 to the curve P_4z_1), whereas second-class solutions (line 3 in Fig. 6) continuously become solutions with a submerged spillway (see Fig. 3). For the second-class solution obtained at $h_0 = h_{02}$, the flows on the drop and behind the drop take identical critical values h_c and v_c that coincide with the constant-flow parameters between the discontinuous s-wave and the depression r-wave in the solution of the classical problem of dam break over a horizontal bottom with the initial data (1.2) at $z_0 = h_0 = h_{02}$. However, this solution, in contrast to the solutions obtained for $h_0 \in (h_{02}, h_{04}]$, is unstable to a slight change in the initial data: for $h_0 = h_{02} - \varepsilon$ ($\varepsilon \ll 1$), the flow behind the drop becomes supercritical, which leads to a sudden transition from hyperbola (5.4) to the shock part of the r-adiabat (6.3), i.e., from the curve P_2P_4 to the curve $P_1P_2'P_3$ in Fig. 4. As a result, this solution becomes a solution with two discontinuous waves on the right of the drop (line 3 in Fig. 5).

Conclusions. In this paper, we consider only those solutions with a loss of the total flow energy on the bottom drop in which three characteristics of system (1.1) arrive at the discontinuity (1.3). At the same time, in the case of real shelves, energy losses take place even in the case of solutions with two characteristics arriving at the discontinuity (1.3). However, as follows from the results of [9], where similar solutions over a bottom drop are studied, accounting for such energy losses, which is possible only on the basis of some heuristic information, leads only to a certain compression of the entire adiabatic diagram (Fig. 4) in the direction of the critical flow line $v = \sqrt{h}$. This, in turn, narrows the regions of existence of nonsubmerged flow regimes with two discontinuous waves and energetically stable flows with three characteristics arriving at the drop. As a result, accounting for such energy losses, though leading to some qualitative changes in the parameters of the flows constructed, does not change the flow pattern drastically. This conclusion has been supported to some extent by the results of laboratory experiments performed V. I. Bukreev and A. V. Gusev, researchers of the Laboratory of Applied Hydrodynamics of the Lavrent'ev Institute of Hydrodynamics, SD RAS, to check the self-similar solutions of the discontinuity decay problem (1.1)–(1.3). The types of waves, their velocities, and asymptotic depths behind the front of the discontinuous s-wave obtained in the present paper agree well with the experimental data.

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